

# C.U.SHAH UNIVERSITY

## Winter Examination-2018

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT2

Branch: Diploma (All)

Semester : 2

Date : 23/10/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1 Attempt the following questions:****(14)**

a)  $\lim_{\theta \rightarrow 0} \frac{\sin 5\theta}{\theta} = \underline{\hspace{2cm}}$

(A) 1 (B) 0 (C) 1/5 (D) 5

b)  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = \underline{\hspace{2cm}}$

(A)  $e^2$  (B)  $e$  (C)  $e^{1/2}$  (D) None of these

c)  $\lim_{x \rightarrow 0} \frac{x^2 + x + 1}{x + 1} = \underline{\hspace{2cm}}$

(A) 1 (B) 0 (C) -1 (D) None of these

d)  $\frac{d(\sqrt{x})}{dx} = \underline{\hspace{2cm}}$

(A)  $\frac{1}{2\sqrt{x}}$  (B)  $\frac{1}{\sqrt{x}}$  (C)  $-\frac{1}{x^2}$  (D)  $2\sqrt{x}$ 

e)  $\frac{d(a^x)}{dx} = \underline{\hspace{2cm}}$

(A)  $a^x \log_e a$  (B)  $\log_e a$  (C)  $x^a \log_e a$  (D)  $a^x \log_e x$ 

f)  $\frac{d(\cos x)}{dx} = \underline{\hspace{2cm}}$

(A)  $-\sec x$  (B)  $\sec x$  (C)  $-\sin x$  (D)  $\sin x$ 

g)  $\frac{d(\cot^{-1} x)}{dx} = \underline{\hspace{2cm}}$

(A)  $\frac{-1}{\sqrt{1-x^2}}$  (B)  $\frac{1}{\sqrt{1-x^2}}$  (C)  $\frac{1}{1+x^2}$  (D)  $\frac{-1}{1+x^2}$ 

h)  $\int 1 dx = \underline{\hspace{2cm}}$



(A) 0 (B) 1 (C)  $x+c$  (D) None of these

i)  $\int \tan x \, dx = \underline{\hspace{2cm}}$

(A)  $\log \cos ecx + c$  (B)  $\log \sec x + c$  (C)  $\log |\cos ecx - \cot x| + c$

(D)  $\log |\sec x + \tan x| + c$

j)  $\int_1^3 \frac{2x}{1+x^2} \, dx = \underline{\hspace{2cm}}$

(A)  $\log 5$  (B)  $\log 2$  (C)  $\log 10$  (D)  $\log 20$

k) If  $\mathbf{x} = (1, 1, 1)$  and  $\mathbf{y} = (1, 0, 0)$  then  $\mathbf{x} - \mathbf{y} = \underline{\hspace{2cm}}$ .

(A)  $(1,0,1)$  (B)  $(0,1,1)$  (C)  $(1,1,0)$  (D)  $(0,0,1)$

l)  $|2i + j - 3k| = \underline{\hspace{2cm}}$ .

(A)  $\sqrt{14}$  (B)  $\sqrt{13}$  (C)  $\sqrt{41}$  (D) 4

m) If  $\bar{x} = 2i - j + k$  and  $\bar{y} = i + 2j$  then  $\bar{x} \cdot \bar{y} = \underline{\hspace{2cm}}$ .

(A) 1 (B) 0 (C) -1 (D) -2

n)  $(2, -1, 3) \times (-4, 2, -6) = \underline{\hspace{2cm}}$ .

(A)  $(-8, -2, -18)$  (B)  $(8, 2, 18)$  (C)  $(0, 0, 0)$  (D) None of these

**Attempt any four questions from Q-2 to Q-8**

**Q-2**

**Attempt all questions**

**(14)**

a) If  $\bar{a} = 2i + j - k$ ,  $\bar{b} = i - j + 2k$  and  $\bar{c} = i - 2j + k$  then find the direction cosines of  $\bar{a} + \bar{b} - 2\bar{c}$ .

**(5)**

b) If  $\bar{a} = i + j + k$  and  $\bar{b} = 2i - 2j + k$  then find unit vector perpendicular to  $\bar{a}$  and  $\bar{b}$ .

**(5)**

c) Evaluate:  $\lim_{x \rightarrow 0} \frac{3^{2x} - 2^{2x}}{x}$

**(4)**

**Q-3**

**Attempt all questions**

**(14)**

a) Evaluate:  $\lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{5}}{\sqrt{x+4} - \sqrt{7}}$

**(5)**

b) Evaluate:  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \sec^2 x}{1 - \tan x}$

**(5)**

c) Find  $\frac{dy}{dx}$  if  $y = e^x \sin x \cos x$ .

**(4)**

**Q-4**

**Attempt all questions**

**(14)**

a)  $S = t^3 - 6t^2 + 8t - 4$  gives the distance travelled by a body in  $t$  seconds. Find velocity and acceleration at  $t = 4$  second.

**(5)**

b) Find  $\frac{dy}{dx}$  if  $y = \log \sqrt{\frac{a+x}{a-x}}$ .

**(5)**

c) If  $\bar{x} = (-4, 9, 6)$ ,  $\bar{y} = (0, 7, 10)$ ,  $\bar{z} = (-1, 6, 6)$  then show that

**(4)**

$(\bar{x} - \bar{z}) \cdot (\bar{y} - \bar{z}) = 0$

**Q-5**

**Attempt all questions**

**(14)**



a) Prove that angle between two vectors  $i + j - k$  and  $2i - 2j + k$  is  $\sin^{-1}\left(\frac{\sqrt{26}}{3\sqrt{3}}\right)$ . (5)

b) Find  $\frac{dy}{dx}$  if  $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ . (5)

c) Evaluate:  $\lim_{x \rightarrow 2} \frac{x^7 - 128}{x^4 - 16}$  (4)

**Q-6**

**Attempt all questions** (14)

a) Forces  $(1, 2, 3)$ ,  $(-1, 2, 3)$  and  $(-1, 2, -3)$  act on a particles and the particle moves from the point  $(0, 1, -2)$  to  $(-1, 3, 2)$ . Find the work done by the forces. (5)

b) Prove that  $\int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x + \cot x} dx = \frac{\pi}{4}$ . (5)

c) Find derivative of  $y = x^3 - 2x$  using first principle. (4)

**Q-7**

**Attempt all questions** (14)

a) Evaluate:  $\int \frac{(1-3x)^2}{x^3} dx$  (5)

b) Find the area of circle  $x^2 + y^2 = r^2$ . (5)

c) For what value of  $R$ , vectors  $2i - 3j + 5k$  and  $Ri - 6j - 8k$  are perpendicular to each other? (4)

**Q-8**

**Attempt all questions** (14)

a) If  $f'(x) = 4x^2 + 6x - 3$  and  $f(1) = 2$  then find function  $f(x)$ . (5)

b) If  $y = A \cos pt + B \sin pt$  then prove that  $\frac{d^2y}{dt^2} + p^2y = 0$ . (5)

c) Evaluate:  $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx$  (4)

